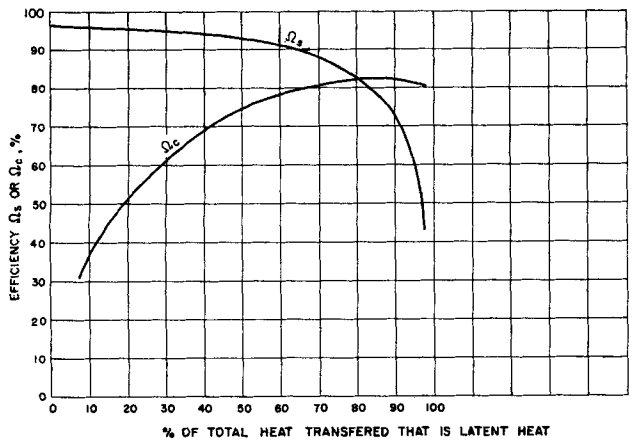

Fig. 4. Variation of fin efficiency with relative rate of condensation, ¾-in. copper fin.

gas such that $p_v > p_i$. When Equation (13) is used, this assumption can be checked by calculating the temperature of the tip or end of the fin by the following expression:

$$t_{end} = \frac{t_b + \alpha/\beta}{\cosh \sqrt{\beta}\, L} - \alpha/\beta \qquad (17)$$

For highly efficient fins, Equation (13) gives overall efficiencies close to those calculated by integration of Equation (10).

CONCLUSIONS

The fin efficiencies for sensible heat transfer and mass transfer will, in general, not be the same. In the numerical example studied here the sensible heat transfer efficiency approached a maximum value when the fraction of the total heat transferred as latent heat approached zero. In the case where the fin end temperature approached the dew point of the gas the condensation efficiency approached zero, while at the same time the sensible heat transfer efficiency approached its maximum value. Based on this study, the rates of heat and mass transfer to fins must be evaluated by use of individual fin efficiencies to obtain their proper values.

NOTATION

a_x = cross-sectional area of fin, sq.ft.
A = area of fin, sq. ft.
h_v = sensible heat transfer coefficient of gas, B.t.u./hr. sq.ft. °F.
k = thermal conductivity of metal, B.t.u./hr. sq.ft. °F./ft.
K_g = mass transfer coefficient, lb. moles/hr. sq.ft. atm.
l = position along fin perpendicular to base, ft.
p = partial pressure or vapor pressure condensable vapor, atm.
Q = heat flow through metal, B.t.u./hr.
S = perimeter of fin, ft.
t = temperature, °F.
T_g = temperature of main body of gas, °F.
w = fin thickness, in.
λ = latent heat of condensation, B.t.u./lb. mole
Ω = thermal efficiency of fin

Subscripts

b = conditions at the base of the fin
c = condensation heat effects
i = condition at gas metal interface
o = overall conditions
s = sensible heat effects
v = vapor

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Note on Dynamics of Liquid-Solid System Expansion and Sedimentation

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Fan, Schmitz, and Miller (*1*) studied the dynamic response of bed height in liquid-solid fluidized bed to a change in fluidizing velocity. Developing previous studies by Slis, Willemse, and Kramers (*2*), these authors derived a linearized analytical model by which they interpreted experimental results. Independently, Massimilla, Volpicelli, and Raso (*3*) studied the properties of pulsing solid-liquid fluidized beds obtained by feeding granular beds with intermittent liquid streams. In this connection, they also investigated a dispersion of dense aggregates of particles in solid-liquid suspensions (*4*).

Fluidization control and pulsing fluidization concern the same problem of unsteady state conditions of a fluidized bed. This is studied by considering two main phenomena, sedimentation of expanded beds and expansion of dense layers of particles.

Solid motion in sedimentation may be regarded as a plug flow, the bed of settled particles being formed piston-like while a suspension is disappearing. System contraction as a function of the time is evaluted with equations for settling of suspension and material balance of liquid and solid (*3, 5, 6, 7, 8, 9*).

Expansion of a granular bed is a more complex phenomenon. Several models have been suggested to interpret solid flow pattern in expansion as a step up input ($\phi - \phi_{ss}$) occurs (*2, 4*). Piston flow and solid complete mixing models are represented in Figures 1*A*, *B*, and *C*. If particles were rigidly connected to each other, the solid phase

would move up as appears in model A, leaving behind clear liquid. Disregarding inertial forces, the velocity of piston rise would be given by equation

$$\frac{dh}{dt} = \phi - U_s\ \epsilon_{ss}^{\ n} \qquad (1)$$

Instability of equilibrium of forces acting on the solid is considered in models B and C. As step up input occurs, disturbances in liquid distribution develop at the bottom of the bed promoting a turbulent diffusion in solid phase. In accordance with model B, instability causes the space behind the plug of dense layers of particles to be filled with a suspension at equilibrium void degree for stepped up fluidizing stream. Model C is based on the concept that solid turbulent diffusion is so active as to assure uniform distribution of particles throughout the solid-liquid system at any time during expansion.

At $t = 0$, the rise velocity of the top of the solid-liquid system is given by Equation (1) in both piston flow and complete mixing models. But, while models A and B assume a constant dh/dt throughout expansion, model C implies a continuous variation of expansion velocity from $(\phi - U_s\ \epsilon^n_{ss})$ at $t = 0$ to 0 at $t = \infty$.

In piston flow, increase of bed height $(h - h_{ss})$ is a linear function of t:

$$h - h_{ss} = (\phi - U_s\ \epsilon^n_{ss})t \qquad (2)$$

for model B this is $0 \leq t \leq \dfrac{h_i - h_{ss}}{\phi - U_s\ \epsilon^n_{ss}}$. In complete mixing flow, step height at time t is

$$h - h_{ss} = \int_0^t \{\phi - U_s\,[\epsilon(t)]^n\}\,dt \qquad (3)$$

In previous work (3), variations in bed height due to step up input were determined by finite difference calculations of the integral in Equation (3). The theoretical approach by Fan, Schmitz, and Miller is also an analytical interpretation of the model of complete mixing of the solid phase. In accordance with model C, they derived the equation

$$\frac{dh}{dt} = \frac{h - h_{ss}}{T} \qquad (4)$$

in which T is a time constant equal to $\dfrac{h_{ss}\,\epsilon_{ss}}{\phi_{ss}\,n(1-\epsilon_{ss})}$ when $(h_i - h_{ss})$ is small compared with h_{ss}. In this case, Equation (4) gives

$$\ln\left[1 - \frac{h - h_{ss}}{h_i - h_{ss}}\right] = -\frac{t}{T} \qquad (5)$$

which is a linearized expression of step height as a function of t approximating exact solution of Equation (3).

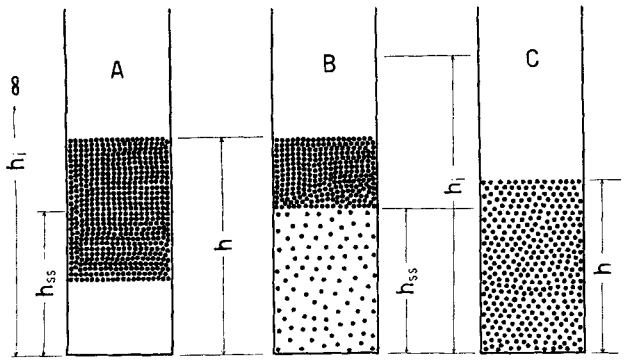

Fig. 1. Expansion of beds of particles. A = Piston flow (rigidly connected particles). B = Piston flow. C = Complete mixing.

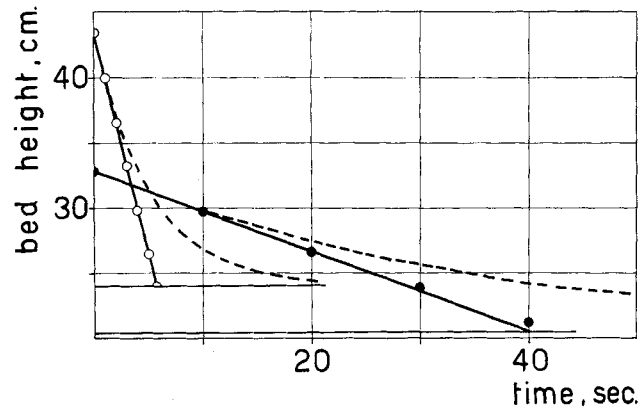

Fig. 2. Sedimentation of beds of nalcite and glass beads. ●— Reference 1, nalcite, 25 to 30 mesh; $\epsilon_{ss} \simeq 0.72$; $h_{ss} = 32.8$ cm.; $h_i = 20.3$ cm.; diameter of the column = 50 mm. ○—Reference 3, glass beads, 20 to 25 mesh; $\epsilon_{ss} = 0.66$; $h_{ss} = 44$ cm.; $h_i = 23.8$ cm.; diameter of the column = 35 mm. ——— Piston flow. - - - - Linearized model for complete mixing according to Equation (5).

Results of experiments carried out by Fan, Schmitz, and Miller (*1*) with fluidized beds of nalcite (25 to 30 mesh) and glass beads (70 to 80 mesh) and by Massimilla, Volpicelli, and Raso (*3*) with fluidized beds of larger glass beads (20 to 25 mesh) are correlated in Figures 2 and 3.

Figure 2 shows that sedimentation data from both series of experiments definitely follow the piston flow model. On the contrary, Figure 3 shows that expansion of a suspension of small glass beads (*1*) follows the complete mixing model, while expansion of a bed of larger glass beads (*3*) follows the piston flow model, at least up to a certain time. Difference in trend of experimental curves when compared with theoretical curves for the two models may be explained by considering difference in degree of stability of beds of small and larger glass beads to a step increase in liquid flow rate. The higher voidage of the system, the lower weight of single particle, and the larger column diameter make integrity of plugs of 70 to 80 mesh glass beads—water system much more sensitive to disturbances than plugs of dense aggregates of larger glass beads. Thus, more readily liquid vortices mix up solid phase causing expansion curve to deviate from piston flow model.

A comparative analysis of diagrams in Figures 2 and 3 outlines that, as far as solid motion is concerned, it is difficult to describe contraction and expansion by a single model.

Fan, Schmitz, and Miller proposed Equation (4) for both phenomena since, in a control problem, $h_i - h_{ss}$ due to fluctuations in liquid flow rate is very small compared with h_{ss}. However, in this case sedimentation and expansion data are equally well correlated in accordance with piston flow and complete mixing models. Also, investigation on pulsing fluidization (*3, 4*) has shown that, generally, expansion itself does not follow only one of the models in Figure 1 but a combination of them. Often, expansion first takes place in accordance with a piston flow model, and then it deviates, approaching more or less closely the complete mixing model. Time required for deviation from piston flow is dependent on the quality and intensity of mixing in the solid phase, and these, in turn, are strictly concerned with instability of equilibrium of forces acting on dense plug of particles as step up in liquid flow rate occurs. More experimental information is necessary on this subject to allow convincing predictions of the expansion pattern of granular beds. So far, it is only possible to confine the actual curves in expansion vs. time

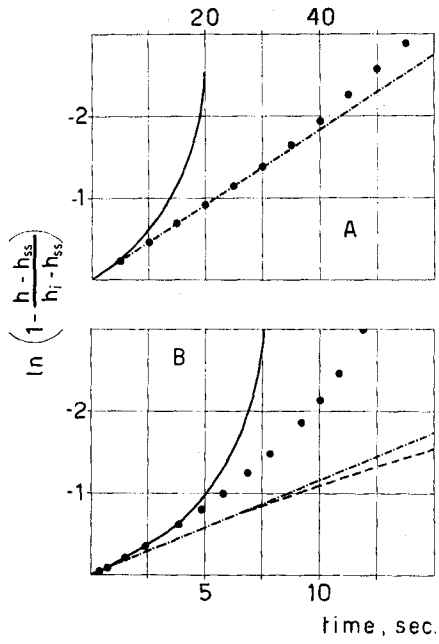

Fig. 3. Expansion of beds of glass beads. A—Reference 1, glass beads, 70 to 80 mesh; $\epsilon_{ss} \simeq 0.66$; $h_{ss} = 14.2$ cm.; $h_i = 26$ cm.; diameter of the column = 50 mm. B—Reference 3, glass beads, 20 to 25 mesh; $\epsilon_{ss} = 0.40$; $h_{ss} = 23.8$ cm.; $h_i = 51.8$ cm.; diameter of the column = 35 mm. ——— Piston flow. - - - - Complete mixing according to Equation (3). ------- Linearized model for complete mixing according to Equation (5).

diagrams within theoretical curves for piston flow and complete mixing models.

NOTATION

h = bed height at time t, cm.
h_i = equilibrium bed height for superficial liquid velocity ϕ, cm.
h_{ss} = equilibrium bed height for superficial liquid velocity ϕ_{ss}, cm.
n = constant in Equation (1), dimensionless
t = time, sec.
T = theoretical time constant, sec.
U_s = terminal falling velocity of the particle, cm./sec.
ϵ = void fraction, dimensionless
ϵ_{ss} = equilibrium void degree for superficial liquid velocity ϕ_{ss}, dimensionless

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On a Generalized Expression for Prediction of Minimum Fluidization Velocity

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One of the important design variables that is encountered in the analysis and design of all fluid bed processes and which is required to be predicted with confidence is the minimum fluidization velocity. At the present time an approximate magnitude of this characteristic can be obtained on the basis of any one out of a few reliable empirical equations currently available (*2, 7, 8, 9, 11, 15*). An excellent review of the better approximation methods has been indicated in the recent work of Leva (*10*). Two approaches are evident in the analysis of the fluidization phenomenon. One of these is concerned with the valid hypothesis that at the point of initial bed expansion all equations pertaining to the behavior of fixed beds apply with equal rigor and that the pressure gradient is balanced by the net weight of the bed. Leva's analysis, which is considered to be one of the best, starts off by coupling the pressure drop equation for laminar flow through a bed of irregular particles with the above identity. The final equation for the prediction of minimum fluidization velocity is obtained on the basis of all available experimental data which establishes the variation of an empirical constant with Reynolds number, thus obviating the necessity to have prior values for bed voidage at minimum fluidization velocity and particle shape factor. Furthermore a correction factor, obtained through a chart (*10a*), is used as a multiplier in the original correlation when the particle Reynolds number exceeds the limit for laminar flow. Wilhelm and Kwauk (*15*) recommend a graphical procedure to be adopted on a plot connecting a dimensionless group $K_{\Delta P}$ (product of friction factor and the square of Reynolds number) and Reynolds number. The rest of the correlations are based on a second approach, namely dimensional analysis. The numerical constants of the final expressions are fixed with the help of experimental data. When a comparison is made of the predicted results from the different correlations for the same particle-fluid system, considerable deviation exists among them, and the extent of deviation appears to be a